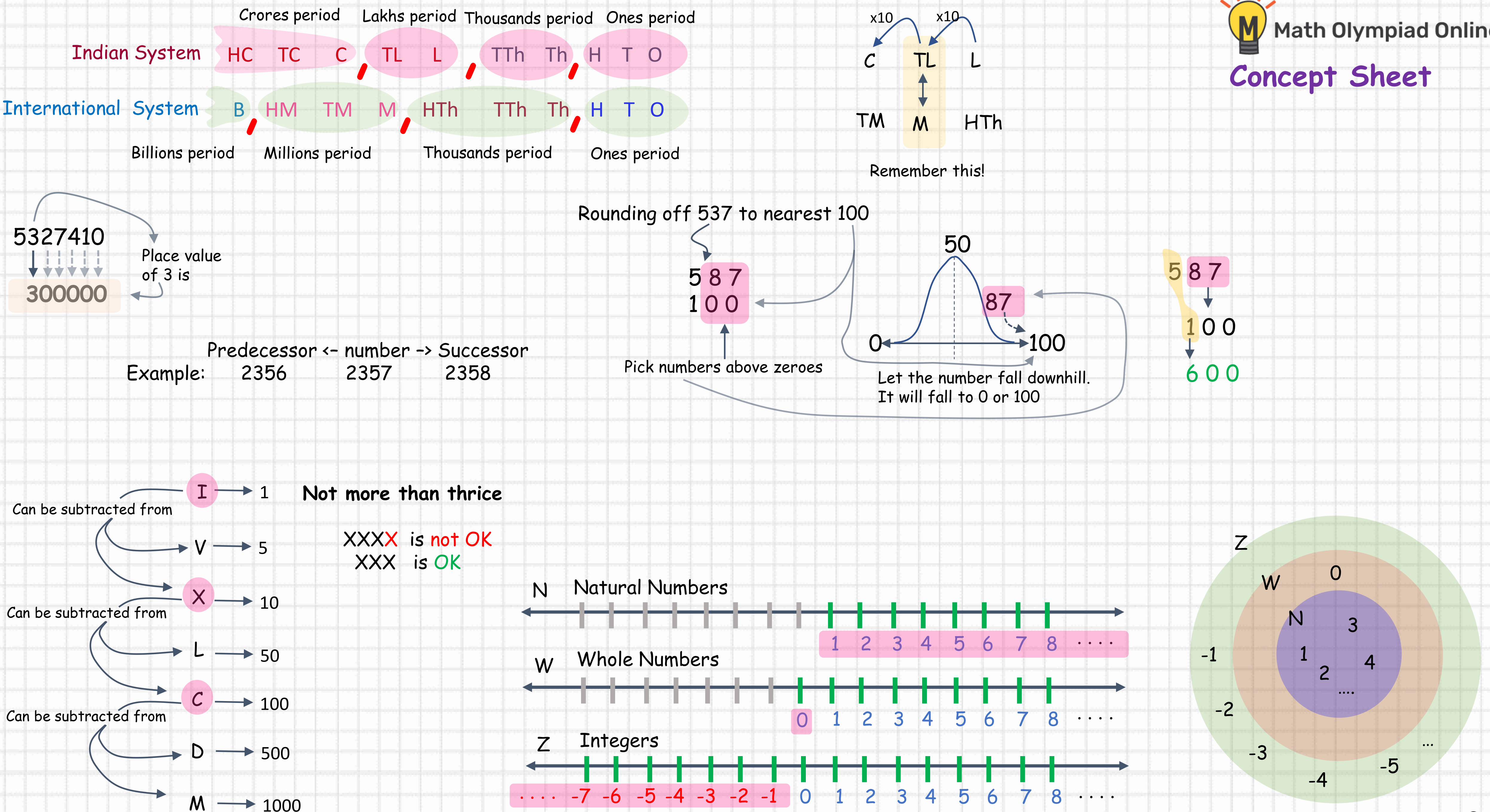
Math Olympiad Online Training

Beginner Course (Grade ~ 5-6)

Concept Sheets

Math Olympiad Online

Concept Sheet



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Concept Sheet

Order of operations

1. ( ) first do operations inside parenthesis
2. × and ÷ (always left to right)
3. + and – (any order is fine)

Properties of numbers

Distributive → Sharing

$a \times (b + c) = a \times b + a \times c$

Associative → Grouping

$(a + b) + c = a + (b + c)$

$(a \times b) \times c = a \times (b \times c)$

Commutative → Movement

$a + b = b + a$ $\quad$ $a \times b = b \times a$

$(a + b) + c = c + (a + b)$

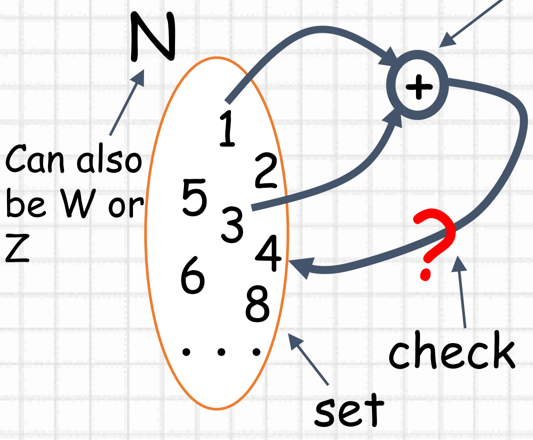
$(a \times b) \times c = c \times (a \times b)$

Zero: 0

1. Neither positive nor negative
2. Is an even number
3. Division by 0 is undefined.

Closure

Can be other operators like – or x or ÷



Operate any two numbers.
If answer always found inside the set, then the set is closed under that operation

Identity

Additive identity : (0)
If you add with this number, value does not change.
Multiplicative identity: (1)
If you multiply with this number, value does not change

Inverse

Additive Inverse:
If you add with this number, you get zero.
Multiplicative Inverse:
If you multiply with this number, you get 1

Integer Arithmetic

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Concept Sheet

Z Integers

.... -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 ....

A number at left < number at right : $-33 < 2$

Negative numbers are used to indicate something decreasing.
It can also be used to indicate direction, usually left side or west side or downside.

Rule for adding/subtracting two numbers

Check how many negative signs

If one negative sign – (subtract)

a) Just subtract the two numbers

b) Put the sign of larger magnitude number

$-7 + 3 = -4$

If two negative sign – - (add)

a) Just add the two numbers

b) Put a negative sign.

$-7 - 3 = -11$

Every Number has a magnitude and a sign.
Sign of a number is the symbol to its left

+3 -5

+ is optional to write

Rule for Multiplication of two numbers

(+)	x	(+)	= +	(3) x (2) = 6
(+)	x	(-)	= -	(3) x (-2) = -6
(-)	x	(+)	= -	(-3) x (2) = -6
(-)	x	(-)	= +	(-3) x (-2) = 6

Rule for Parenthesis same as multiplication

3 + (+4) = 3 + 4
3 + (-4) = 3 - 4
3 - (+4) = 3 - 4
3 - (-4) = 3 + 4

Note: -2 +4 -7 -5 +3 = ?

You can choose to group any two numbers, but remember to take the sign along with the number

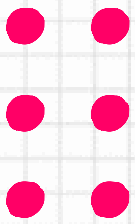
$\boxed{-2\ +4}$ -7 -5 +3
$\boxed{2\ \ -7}$ -5 +3
$\boxed{-5\ \ -5}$ +3
$\boxed{-10\ +3}$
-7 ✓

-2 +4 -7 - $\boxed{5\ +3}$
-2 +4 -7 - 8 ✗

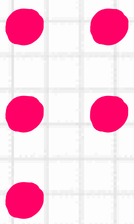
Grouping this way is wrong because we left out the sign of number 5
We should have grouped as $\boxed{-5+3}$ = -2

Playing with Numbers

Even and Odd Numbers



Even numbers come in pairs of dots

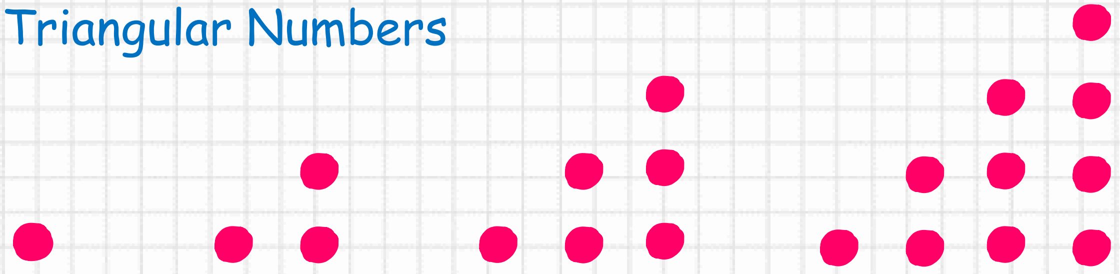


Odd numbers will have a lone dot

Even + Even = Even
Even + Odd = Odd
Odd + Odd = Even

Even x Even = Even
Odd x Even = Even
Odd x Odd = Odd

Triangular Numbers



1   3=1+2   6=1+2+3   10=1+2+3+4

n^{th} triangular number = $\frac{n\,(n+1)}{2}$

Example: 5^{th} triangular number = (5*6)/2 = 15

1+2+3+4+5+ …… +10 = (10*11)/2 = 55

Using distributive property

2+4+6+8+10+…… +20 = $\boxed{2*}$1+$\boxed{2*}$2+$\boxed{2*}$3+$\boxed{2*}$4+$\boxed{2*}$5+ …… +$\boxed{2*}$10= $\boxed{2*}$(1+2+3+4+5+ …… +10)=2*(10*11)/2 = 110

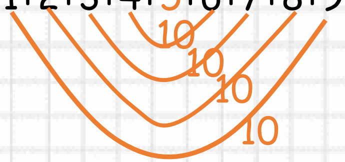
Handshake problem: 5 people, how many handshakes possible if each person handshakes only once with another person?
Answer: (5-1)th triangular number = 4^{th} triangular number = 4*5/2=10

Chess game: 9 kids, how many games possible if each kid plays only once with another kid ?
Answer: (9-1)th triangular number = 8^{th} triangular number = 8*9/2=36

Another method to add consecutive numbers

1+2+3+4+5+6+7+8+9 = ?

1+2+3+4+5+6+7+8+9   = 10+10+10+10+5 = 45



Playing with Numbers

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Concept Sheet

Square Numbers

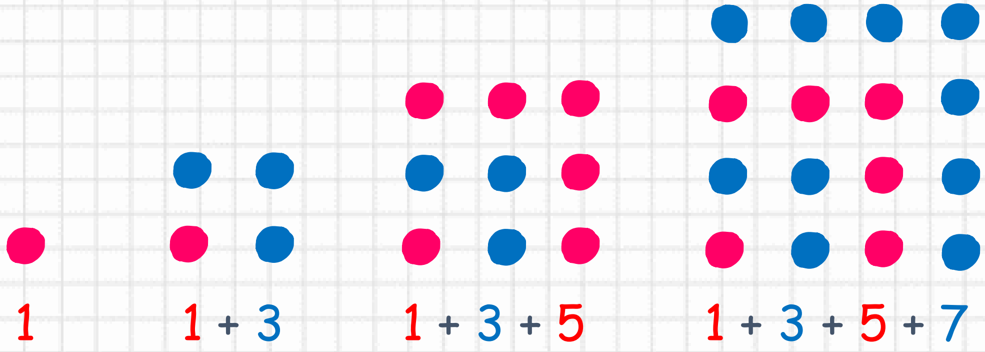
1x1, 2x2, 3x3, 4x4 etc

1+3 = 4

1+3+5 = 9

1+3+5+7 = 16

1+3+5+7+9 = 25



Adding first 5 odd numbers equals 5th square

Adding first 8 odd numbers equals 8th square and so on.

1+3+5+7+ …… +21 = ?

First, find how many numbers are there

You don't have to count. Easy method

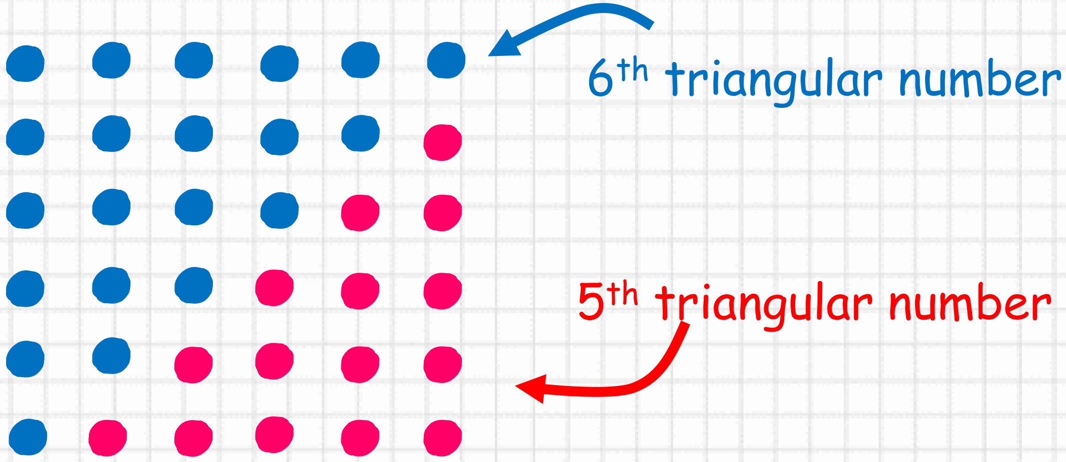
How many numbers ?

= (last number + 1) /2 = (21+1)/2 = 11.

So there are 11 numbers. Then,

1+3+5+7+ …… +21 = 11th square = 11*11=121

Connection between Triangular Number and Square Number



Adding 5th and 6th triangular numbers gives 6th square number

Example: IF you want to find 15th square number, you can add 15th and 14th triangular numbers = (15*16)/2 + (14*15)/2

= 120 + 105

= 225

This is same as doing 15x15

Playing with Numbers

Prime Numbers

Building blocks. All other numbers can be created with prime numbers
All numbers that are not prime are called composite numbers.

What are Prime Numbers ?
Numbers that have only 2 factors: 1 and itself.
What are Composite Numbers?
Numbers that are not Prime. That is, numbers that have more than 2 factors.

1 – Neither prime nor composite

2 – Only prime number that is even.

How to know all prime numbers from 1 to 100 ?

You know 2 , 3 are prime numbers

Then, write all numbers on either side of multiples of 6

5,6,7 11,12,13 17,18,19 23,24,25 29,30,31 35,36,37 41,42,43 47,48,49

53,54,55 59,60,61 65,66,67 71,72,73 77,78,79 83,84,85 89,90,91 95,96,97

Then strike out numbers that you know are divisible by 5 and 7

5,6,7 11,12,13 17,18,19 23,24,~~25~~ 29,30,31 ~~35~~,36,37 41,42,43 47,48,~~49~~

53,54,~~55~~ 59,60,61 ~~65~~,66,67 71,72,73 ~~77~~,78,79 83,84,~~85~~ 89,90,~~91~~ ~~95~~,96,97

You may have to remember that 91 is divisible by 7

Numbers in green are your prime numbers!!

Relatively prime, Mutually prime OR Co-prime :
Two numbers are co-prime if they have only 1 as their common factor.
Example: 14 and 15 are co-prime. Why ?.
14 : factors are 1,2,7,14
15 : factors are 1,3,5,15

- Any two consecutive numbers are always co-prime.
- Any two numbers are co-prime if one or both numbers are itself a prime number.

Twin Prime:
Two prime numbers that are consecutive odd numbers.
Example: 11 and 13.

How to tell if a number is prime ?. – Check if it is divisible by any number from 2 up to half the given number.
How to tell if a number is not prime ?
a) Check if it is of the form 6n-1 or 6n+1. If it is not, then it is not a prime number.
b) If it is of the form 6n-1 or 6n+1, check if it is divisible by any number (Using divisibility test). If it is divisible by some number other than 1 and itself, it is not prime.

Multiples

3: 3, 6, 9, 12, 15, 18, 21, 24 .....
4: 4, 8, 12, 16, 20, 24 .....

12 is the lowest common multiple(LCM).
Other common multiples are simply multiples of LCM!!

Factors

Factors of 24: 1,2,3,4,6,8,12,24
Factors of 16: 1,2, 4, 8, 16

8 is the highest common factor(HCF)
Other common factors are simply factors of HCF!!

LCM Method: What is LCM of 12,16 and 20

Use any number

2	12,16,20
2	6 ,8 , 10
	3 ,4 , 5

LCM=2x2x3x4x5 = 240

HCF Method: What is HCF of 12,16 and 20

Use only prime

2	12
2	6
	3

2	16
2	8
2	4
	2

2	20
2	10
	5

Do Prime factorization

12=$\boxed{2x2}$x3
16=$\boxed{2x2}$x2x2
20=$\boxed{2x2}$x5

HCF = 2x2 = 4

Relation between LCM and HCF of two numbers

Product of two numbers = LCM X HCF

Note:
1) If two numbers are co-prime, HCF = 1, so Product of two numbers = HCF x LCM = 1 x LCM = LCM
2) HCF is always a factor of LCM

Divisibility Test

÷ by 2 : Number ending in 0,2,4,6,8 (all even numbers) are divisible by 2.

÷ by 3 : Add all digits. If the sum is divisible by 3, then the number is divisible by 3.

÷ by 4 : If last two digits of the number are divisible by 4, then the number is divisible by 4.

÷ by 5 : If number ends in 0 or 5, it is divisible by 5.

÷ by 6 : If number is divisible by 2 AND divisible by 3, then it is also divisible by 6 (LCM of 2 and 3 is 6)

÷ by 7 : Cut out last digit , x by 2. Subtract product from rest of number. If answer is still big number, repeat the same method for the answer until you get an answer for which you can tell if divisible by 7 or not. If answer divisible by 7, then the given number is divisible by 7.

Divisibility Test

÷ by 8 : If number formed by last three digits of a given number is divisible by 8, then the given number is divisible by 8.

÷ by 9 : Sum up all digits. If the sum is divisible by 9, then the number is divisible by 9.

÷ by 10 : All numbers ending in 0 are divisible by 10.

÷ by 11 : Separate out the alternate digits into two groups and add them up separately. If the sum comes out same in both the groups, it is divisible by 11. If not, if difference is multiple of 11, then also divisible by 11.

÷ by 12 : If a number is both divisible by 3 AND divisible by 4, then it is divisible by 12. (since LCM of 3 and 4 is 12).

Note:

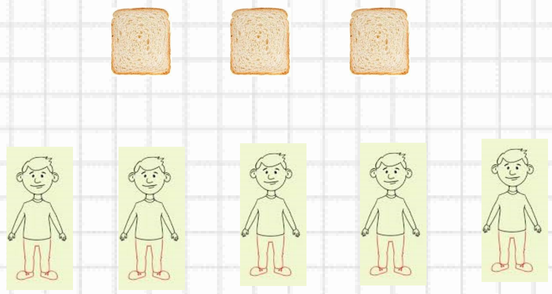
If a number "N" is divisible by number "a" and number "b", then the number "N" is also divisible by LCM of a and b.

Example: 90 is divisible by 3 and 5. Hence 90 must also be divisible by LCM(3,5). So 90 is also divisible by 15.

Another way to say the same fact : If "a" and "b" are factors of a number "N", then LCM(a,b) must also be a factor of number "N".

Fraction as a "Sharing Problem"

$$3 \div 5$$



$$\frac{3}{5}$$

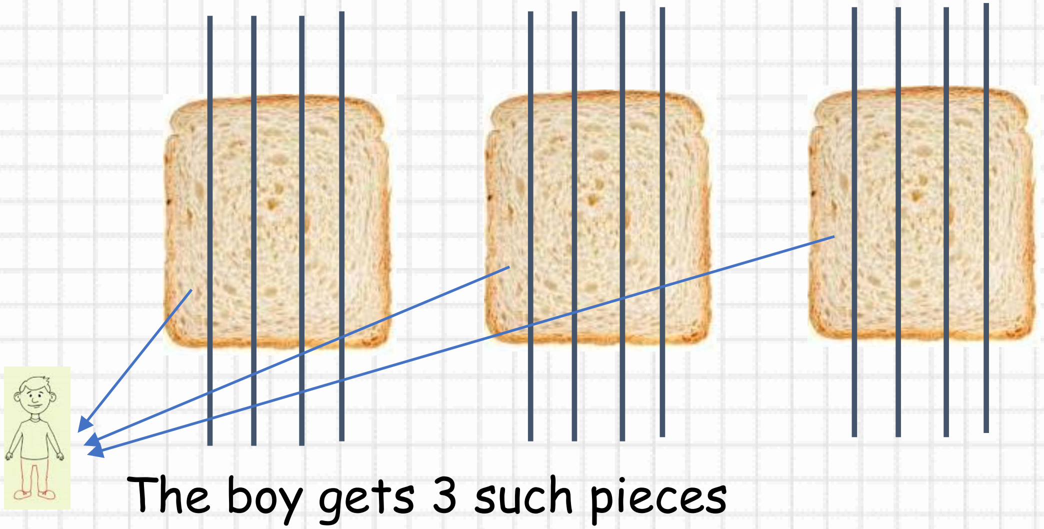
3 whole pieces of bread

equally shared among 5 kids

$$\frac{3}{5}$$

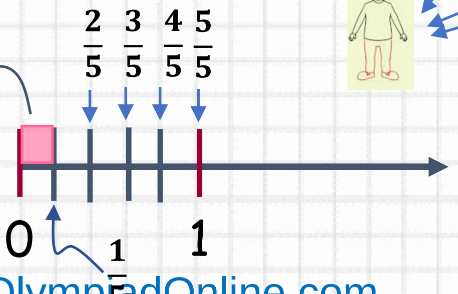
Take each bread and make 5 equal pieces. Repeat for all 3 breads

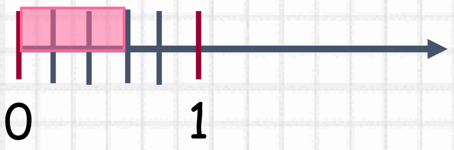
$$3 \, x \, \frac{1}{5}$$



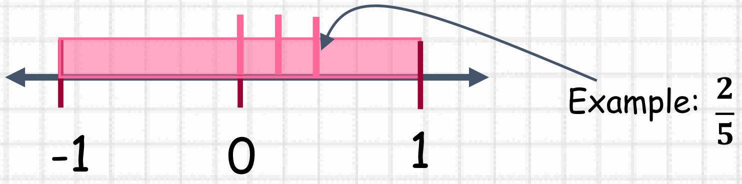
The boy gets 3 such pieces

1 whole bread divided into 5 pieces. This is one such piece.

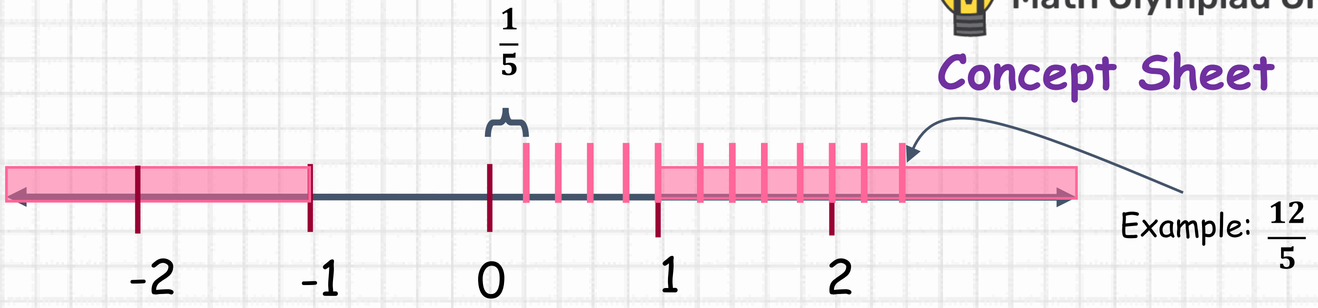




Proper, Improper and Mixed Fractions

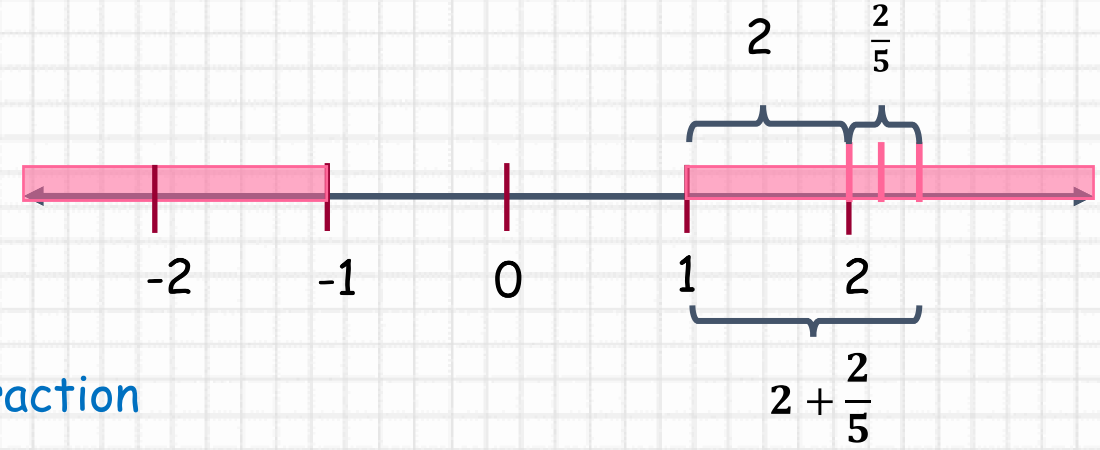


Value between -1 and 1 ➔ Proper fraction



Value outside of -1 and 1 ➔ Improper fraction

This can also be written as a whole number plus a fraction



The "+"sign shall be omitted. $2 + \frac{2}{5}$ = $2\frac{2}{5}$

This is called a Mixed Fraction

Converting Mixed Fraction to Improper Fraction

$$2\frac{2}{5} = \frac{5 \; x \; 2 + 2}{5} = \frac{12}{5}$$

Comparing two fractions

Which is greater ?. $\frac{3}{4}$ or $\frac{4}{6}$

First, make either the Denominator or Numerator the same

Numerator

$\frac{3}{4}$ $\frac{4}{6}$

Denominator

How to change Numerator or Denominator without changing the value of the fraction?

$$\frac{3}{4} = \frac{3xa}{4xa}$$

If you multiply both numerator and Denominator with some number "a", the value of the fraction remains same

Method #1 : Making Denominator the same

$\frac{3}{4}$ $\frac{4}{6}$

Multiply 4 with some number and multiply 6 with some other number so that they both become a new value that is same.

4: 4 8 12 16 20 24 28 32

6: 6 12 18 24 30

The smallest value that is a common multiple for both 4 and 6 is 12.
But we can also choose the product 6x4 = 24.

$$\frac{3}{4} = \frac{3x3}{4x3} \qquad \frac{4}{6} = \frac{4x2}{6x2}$$

Same

$$\frac{9}{12} > \frac{8}{12}$$

So $\frac{3}{4} > \frac{4}{6}$

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Concept Sheet

Comparing two fractions

Which is greater ?. $\frac{3}{4}$ or $\frac{4}{6}$

Method #2 : Making Numerator the same

$\frac{3}{4}$ $\frac{4}{6}$

Multiply 3 with some number and multiply 4 with some other number so that they both become a new value that is same.

We shall choose LCM(3,4) or the product 3x4 as the common value. Here both the LCM and product are the same (because 3 and 4 are co-prime !)

Same

$$\frac{3}{4} = \frac{3x4}{\underbrace{4x4}} \rightarrow \frac{12}{16} \qquad \frac{4}{6} = \frac{4x3}{\underbrace{6x3}} \rightarrow \frac{12}{18}$$

12 pieces of bread given to 16 kids

12 pieces of bread given to 18 kids

The kids among 16 will get more share than the kids among 18. So

$$\frac{12}{16} > \frac{12}{18}$$

So $\frac{3}{4} > \frac{4}{6}$

Note: Once numerator is made the same, the fraction with larger denominator is the smaller value.

Comparing two fractions

Which is greater ?. $\frac{3}{4}$ or $\frac{4}{6}$

Method #3: Cross multiply – easiest method

$$\frac{3}{4} \quad \frac{4}{6}$$

3x6 4x4

18 > 16

Left side > Right side.

$$\frac{3}{4} > \frac{4}{6}$$

Fraction Arithmetic

#1 $\frac{a}{a} = 1$

#2 $\frac{a}{1} = a$

#3 $x \times \frac{a}{b} = \frac{x \times a}{b}$

#4 $\frac{a}{b} = \frac{x \times a}{x \times b}$, $\frac{a}{b} \times b = a$

#5 $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

#6 $\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{d \times b} = \frac{a \times d + c \times b}{b \times d}$

To make denominator same, you can either take LCM(b,d) or simply do bxd. Here we do bxd so calculation becomes easy.

#7 $x + \frac{a}{b} = x\frac{a}{b} = \frac{b \times x + a}{b}$

#8 $\frac{1}{\frac{a}{b}} = \frac{b}{a}$, $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$

Write Numerator as is. Take reciprocal of Denominator and multiply.

Example:

$$\frac{1}{4} + \frac{2}{3} - \frac{6}{7} + \frac{4}{9} + 3\frac{1}{2} + \frac{\frac{1}{3}}{2\frac{1}{3}}$$

Step 1: Simplify the fractions first.

You can separate the whole number and later add to final answer
(OR)
Make this mixed fraction into improper fraction

$$3\frac{1}{2} = \frac{7}{2}$$

$$\frac{\frac{1}{3}}{2\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{7}{3}} = \frac{1}{3} \times \frac{3}{7} = \frac{1}{7}$$

Make denominator an improper fraction

Keep Numerator as is. Write reciprocal of Denominator and multiply

$$\frac{1}{4} + \frac{2}{3} - \frac{6}{7} + \frac{4}{9} + \frac{7}{2} + \frac{1}{7}$$

Step 2: Take LCM of all denominator numbers to make denominator the same

LCM(4,3,7,9,2,7) = 4X9X7= 252

Example:

Step 3: Find how much to multiply each denominator term to get LCM. Multiply that value with numerator

$$\frac{1}{4} + \frac{2}{3} - \frac{6}{7} + \frac{4}{9} + \frac{7}{2} + \frac{1}{7}$$

$252 \div 4 = 63$

$$\frac{63x1 + 84x2 - 36x6 + 28x4 + 126x7 + 36x1}{252} = \frac{1477}{252}$$

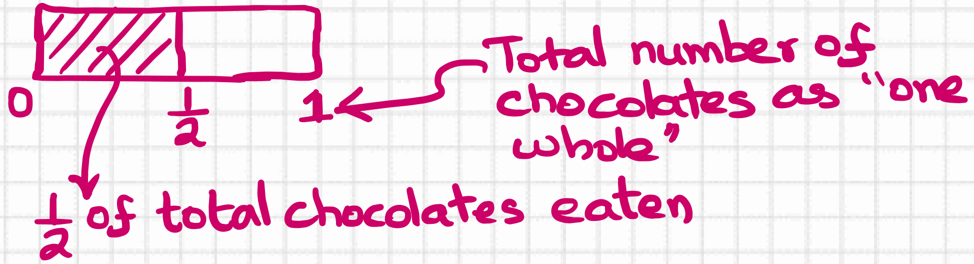
LCM

Fraction as relative vs Absolute

Fraction as "relative": Example: I ate half of my chocolates.

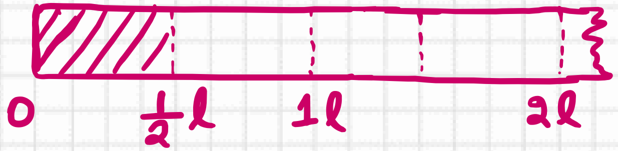
Here, $\frac{1}{2}$ is a "relative" term.
The total number of chocolates is considered as "one whole"

Note: When fraction appears in relative usage, no unit (Kg, l, m, h etc.) will appear



Fraction as "absolute": Example: I drank half litre of water.

Here $\frac{1}{2}$ refers to an absolute value directly on the number line



This is absolute quantity no matter how much total water was available to drink.

Note: When fraction appears in absolute usage, it will always come with a unit (in some cases it can just be a number indicating the count of a thing)

Fraction as relative vs Absolute

Moving between relative and absolute values:

We often need to convert a relative usage to absolute quantity. We also often need to convert an absolute quantity to a relative fraction. How to make such conversions ?

Relative to Absolute Conversion:

I had 12 chocolates. I add $\frac{3}{4}$th of the chocolates. How many chocolates did I eat ?

means "x"

no unit ⇒ relative

$$\frac{3}{4} \times 12 = 9 \text{ chocolates}$$

relative

Absdute

Relative fraction X Total = Absolute

Fraction as relative vs Absolute

Absolute to Relative Conversion:

I had 20 chocolates. I gave 12 chocolates to my friend. What fraction of my chocolates did I give to my friend?

20 chocolates

friend 12

remaining 8

Absolute

Relative

$\frac{12}{20} = \frac{3}{5}$ of my chocolates

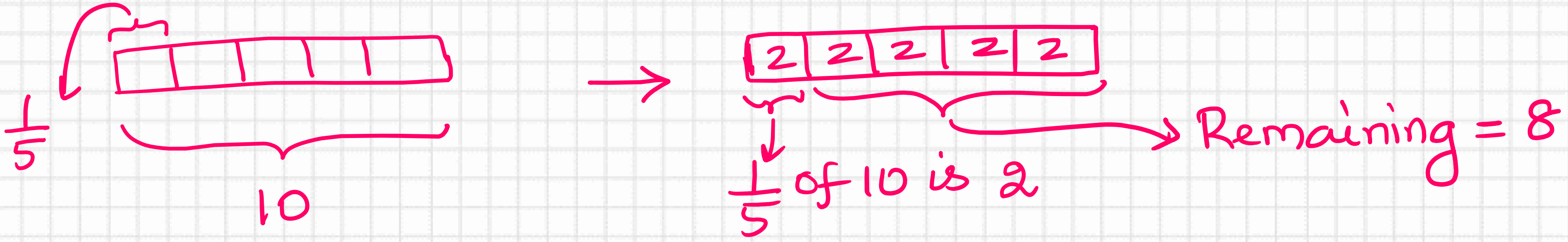
Total chocolates

Absolute ÷ Total = Relative fraction

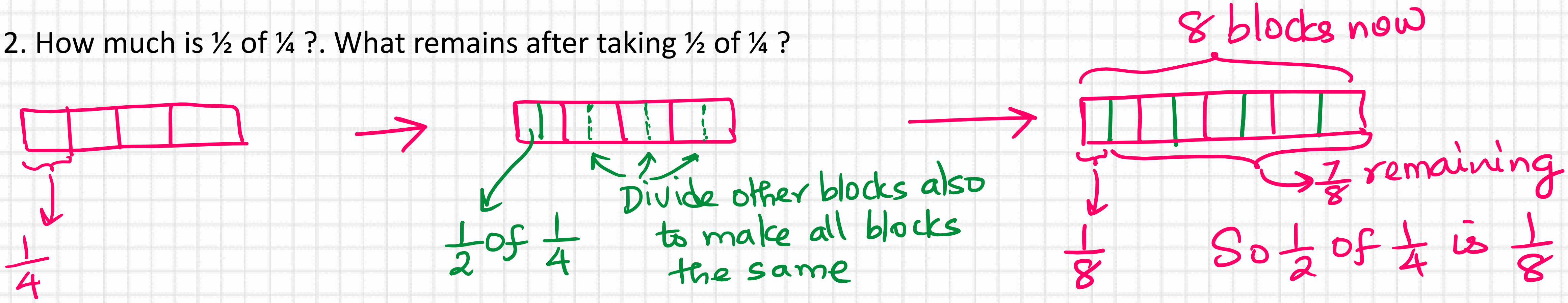
Fraction word Problems

Method #1 : Bar Model Method

1. How much is 1/5 of 10 ?. What remains after taking 1/5 of 10 ?.



2. How much is ½ of ¼ ?. What remains after taking ½ of ¼ ?

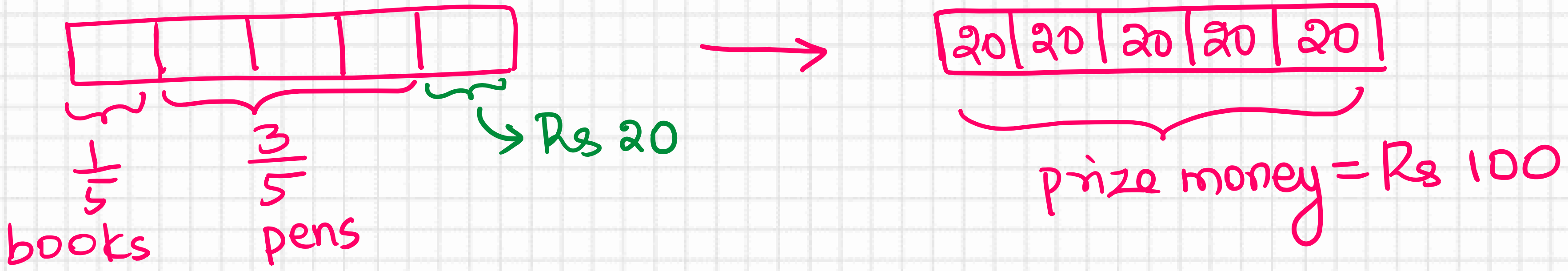


After taking $\frac{1}{2}$ of $\frac{1}{4}$ we have $\frac{7}{8}$ remaining

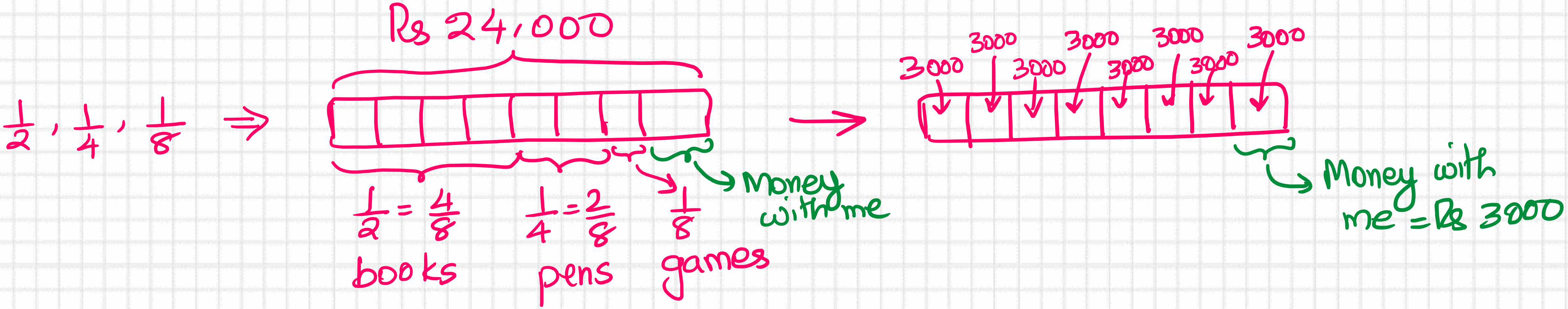
Fraction word Problems

Method #1 : Bar Model Method

3. I spent 1/5 of my prize money on books and 3/5 on pens. If I had Rs 20 left with me, how much was my prize money ?



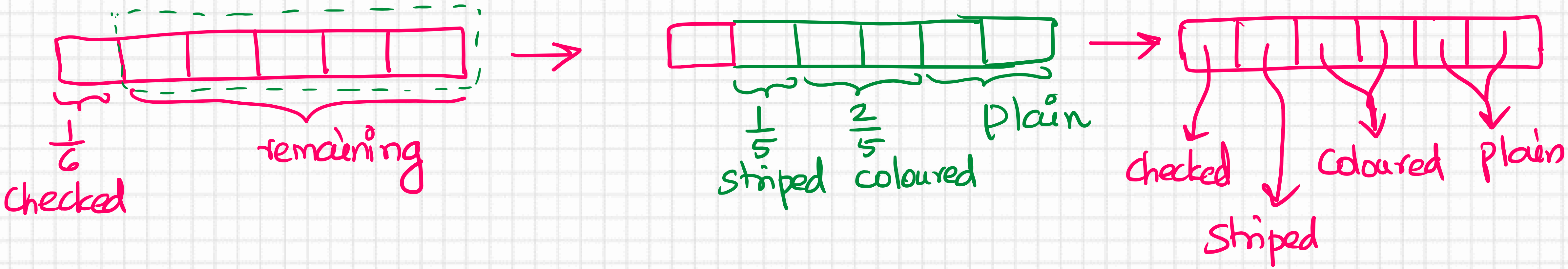
4. I got a salary of Rs 24000. I spent 1/2 of my salary on books, ¼ on pens, 1/8 on games. How much do I have with me now ?



Fraction word Problems

Method #1 : Bar Model Method

5. Among the shirts I have, 1/6 of them are checked. In the remaining shirts, 1/5 of them are striped and 2/5 of them are coloured. The remaining are plain. What fraction of my shirts are plain ?.



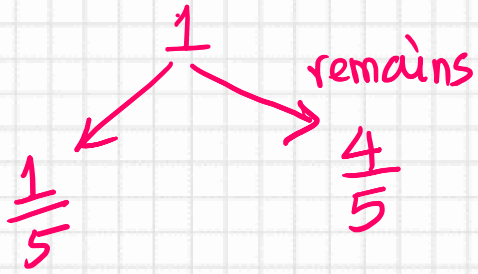
$\text{plain} = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3}$rd of my shirts are plain

Fraction word Problems

Method #2 : Tree Method

1. How much is 1/5 of 10 ?. What remains after taking 1/5 of 10 ?.

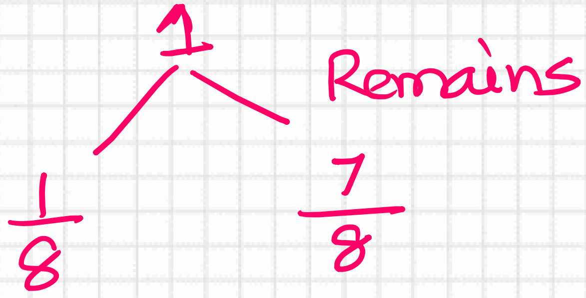


$\frac{1}{5}$ of $10 = \frac{1}{5} \times 10 = 2$

Remaining $= \frac{4}{5}$ of $10 = \frac{4}{5} \times 10 = 8$

2. How much is ½ of ¼ ?. What remains after taking ½ of ¼ ?

$\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$



Remaining $= \frac{7}{8}$

Fraction word Problems

Method #2 : Tree Method

3. I spent 1/5 of my prize money on books and 3/5 on pens. If I had Rs 20 left with me, how much was my prize money ?

1

Books $\frac{1}{5}$ | Pens $\frac{3}{5}$ | Remaining $1-\frac{4}{5}=\frac{1}{5}$

Total $=\frac{4}{5}$

$\frac{1}{5}$ of prize money = Rs 20

$\frac{1}{5}\times$ prize money = Rs 20

So prize money = Rs 20×5 = Rs 100 //

4. I got a salary of Rs 24000. I spent 1/2 of my salary on books, ¼ on pens, 1/8 on games. How much do I have with me now ?

1

Books $\frac{1}{2}$ | Pens $\frac{1}{4}$ | Games $\frac{1}{8}$ | Remaining $1-\frac{7}{8}$ $=\frac{1}{8}$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{4+2+1}{8}=\frac{7}{8}$

I now have $\frac{1}{8}$ of salary

$=\frac{1}{8}\times 24{,}000$

= Rs 3,000 //

Fraction word Problems

Method #2 : Tree Method

5. Among the shirts I have, 1/6 of them are checked. In the remaining shirts, 1/5 of them are striped and 2/5 of them are coloured. The remaining are plain. What fraction of my shirts are plain ?.

1

Checked $\frac{1}{6}$

Remaining $= 1 - \frac{1}{6} = \frac{5}{6}$

Striped $\frac{1}{5} \times \frac{5}{6} = \frac{1}{6}$

Coloured $\frac{2}{5} \times \frac{5}{6} = \frac{2}{6}$

$\underbrace{\frac{1}{6} \qquad \frac{2}{6}}_{\text{Total} = \frac{3}{6}}$

Remaining is Plain $= \frac{5}{6} - \frac{3}{6} = \frac{2}{6}$

Fraction of my shirts that are plain $= \frac{2}{6} = \frac{1}{3}$

Decimal Numbers

$\dfrac{a}{b}$ → When the denominator becomes a multiple of 10, there is a special way to write this fraction using our Decimal Number System

Indian System	HC	TC	C	,	TL	L	,	TTh	Th	,	H	T	O
International System	B	,	HM	TM	M	,	HTh	TTh	Th	,	H	T	O

Dot → Decimal point

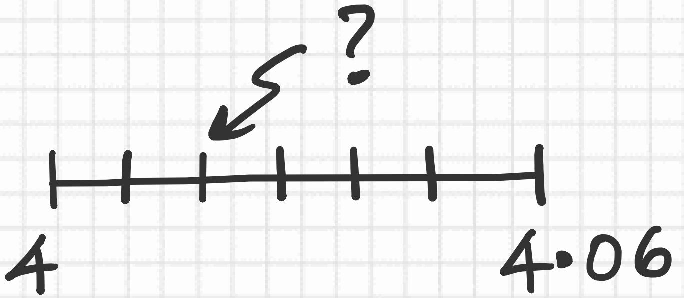
B	HM	TM	M	HTh	TTh	Th	H	T	O	.	Tth	Hth	Thth	
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10	1		$\frac{1}{10}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10000}$ ...

Tenths Hundredths Thousandths

10^2 read as 10 power 2

2 indicates number of times 10 is multiplied with itself

Marking Decimal Numbers on a Number Line



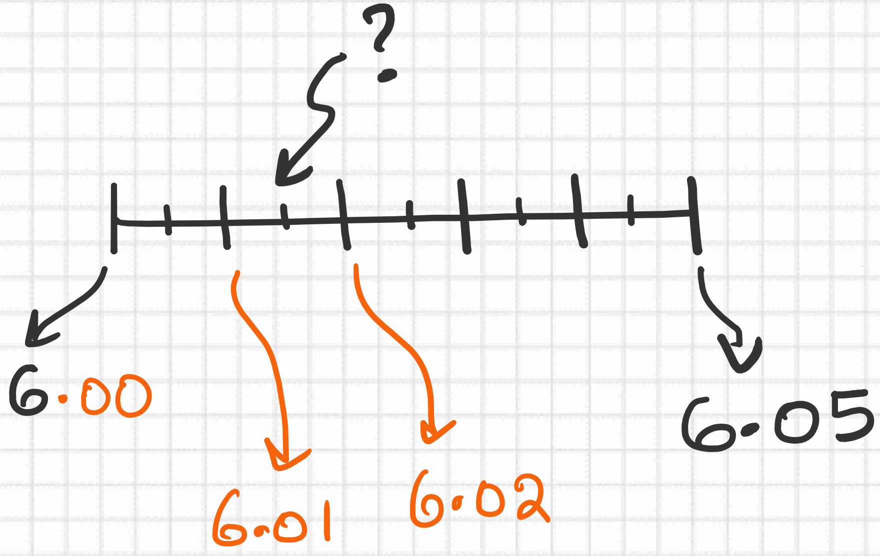
First make the two numbers have same number of decimal digits.

4.00 and 4.06

Next ignore decimal point and simply note numbers inbetween

4.01, 4.02, 4.03, 4.04, 4.05, 4.06

? = 4.02



Now, we cannot write any number between 6.01 and 6.02 by ignoring decimal point. In that case, add an extra zero

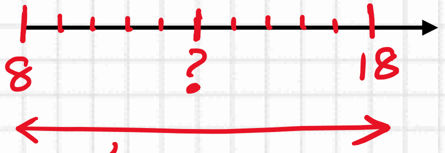
6.010 and 6.020.

Now 6.010, 6.011, 6.012, - - - - 6.019, 6.020

The middle number is 6.015 //

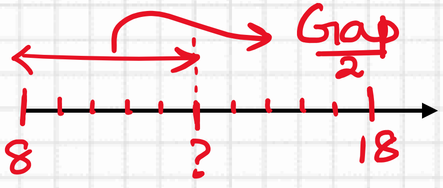
Marking Decimal Numbers on a Number Line

1



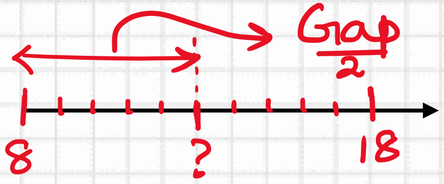
$$\text{Gap} = 18 - 8 = 10$$

⇒



$$\frac{\text{Gap}}{2} = \frac{18-8}{2} = 5$$

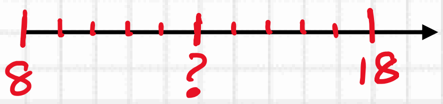
⇒



$$8 + \frac{\text{Gap}}{2} = 8 + \frac{18-8}{2} = 8 + 5 = 13$$

2 A formula can be used to find the middle value:

$$\text{Mid value} = \frac{\text{Start Value} + \text{End Value}}{2}$$



$$\frac{8+18}{2} = \frac{26}{2} = 13$$

The above two methods can also be used to find an unknown value on a number line.

Fraction to Decimal Conversion

Method #1 : Make denominator a multiple of 10.

$$\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10} = 0.6$$

There are six tenths

$$\frac{7}{20} = \frac{7\times 5}{20\times 5} = \frac{35}{100} = 0.35$$

There are thirty five hundredths
OR
There are three tenths and five hundredths

$$\frac{32}{5} = 6\frac{2}{5} = 6 + \frac{2\times 2}{5\times 2} = 6 + \frac{4}{10} = 6 + 0.4$$
$$= 6.4$$

Note: Sometimes, we cannot make Denominator a multiple of 10. We will study such numbers later.

Fraction to Decimal Conversion

Method #2 : Long Division

$$\frac{3}{5} \longrightarrow \begin{array}{r} 0 \\ 5\overline{)\,3} \\ \underline{0} \\ 3 \end{array} \longrightarrow \begin{array}{r} 0.6 \\ 5\overline{)\,3.0} \\ \underline{0} \\ 3\,0 \\ \underline{3\,0} \\ 0 \end{array} = 0.6$$

$$\frac{32}{5} \longrightarrow \begin{array}{r} 6 \\ 5\overline{)\,32} \\ \underline{30} \\ 2 \end{array} \longrightarrow \begin{array}{r} 6.4 \\ 5\overline{)\,32.0} \\ \underline{30} \\ 2\,0 \\ \underline{2\,0} \\ 0 \end{array} = 6.4$$

Decimal to Fraction Conversion

$$0.2 = \frac{2}{10}$$

$$0.75 = \frac{75}{100} = \frac{3 \times 25}{4 \times 25} = \frac{3}{4}$$

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

$$0.125 = \frac{125}{1000} = \frac{25 \times 5}{25 \times 40} = \frac{5}{40} = \frac{1}{8}$$

Decimal to Fraction is simple!

Decimal Number Arithmetic

Addition/Subtraction

$2.3 + 2.45 = ?$ $\longrightarrow$

Align the decimal point

$$\begin{array}{r} 2.30 \\ +\ 2.45 \\ \hline 4.75 \\ \hline \end{array}$$

$2.009 + 1.139 = ?$ $\longrightarrow$

$$\begin{array}{r} 2.009 \\ +\ 1.139 \\ \hline 3.148 \\ \hline \end{array}$$

Decimal Number Arithmetic

Multiplication

① Note down the number of digits after decimal point and add them

$$\underbrace{2.\underbrace{56}_{2}}\times \underbrace{3.\underbrace{1}_{1}}$$

$2+1=3$

② Just multiply the two numbers ignoring decimal point.

$$\begin{array}{r} 256 \\ \times\ 31 \\ \hline 256 \\ 768 \\ \hline 7936 \\ \hline \end{array}$$

Put decimal point such that we get 3 decimal digits

$7.\underbrace{936}$ 3 decimal digits.

Decimal Arithmetic

Division

First try decimal number ÷ whole

$32.5 \div 5$

$$
\begin{array}{r}
6.5 \\
5\overline{)32.5} \\
\underline{30} \\
2\,5 \\
\underline{2\,5} \\
0
\end{array}
$$

These two should have same number of decimal digits.

If remainder does not become zero, keep adding zeroes to the dividend

Decimal Arithmetic

Division

Next : Dividing decimal number by another decimal number.

$32.565 \div 1.25$

$$\frac{32.565 \times 100}{1.25 \times 100} = \frac{3256.5}{125} \rightarrow$$ Now divide as before.

This is to make it whole number

Algebra

Finding the Unknown :

Step #1 Use symbols to indicate unknown quantity.

Step #2 Write your problem as an equation

Step #3 Keep your unknown symbol on one side. Move everything else to other side of the equation.

The value of unknown quantity is now solved.

Example: I have some money. If I add 5 to it and then multiply it by 10, I get 100. How much money do I have ?

I have some money. If I add 5 to it and then multiply it by 10, I get 100. How much money do I have ?

Unknown money "x"

Step#1

$x \rightarrow \boxed{+5} \rightarrow x+5$

$x+5 \rightarrow \boxed{\times 10} \rightarrow \underbrace{(x+5)\times 10 = 100}_{\text{This is an equation}}$

$(x+5)\times 10 = 100$

Step#2

Finding the Unknown :

Step #3 Keep your unknown symbol on one side. Move everything else to other side of the equation.
The value of unknown quantity is now solved.

How to move numbers and expressions in an equation ?

$A + B = C$

Encircle terms such that there is only one operator ('+' or '−' or 'x' or '÷') seen between the circles

$A = C - B$

sign change

keep your unknown symbol (A) on one side and move everything else to other side

$A - B = C$

$A = C + B$

sign change

$-A + B = C$

$B = C + A$

sign change

$A - B = C$

$-B = C - A$

sign change

Algebra

How to move numbers and expressions in an equation ?

$$A \times B = C$$

$$A = \frac{C}{B}$$

× becomes ÷

$$\frac{A}{B} = C$$

$$A = C \times B$$

÷ becomes ×

$$A \times -B = C$$

$$A = \frac{C}{-B}$$

× becomes ÷

$$\frac{A}{B} = C$$

$$\frac{1}{B} = \frac{C}{A}$$

× becomes ÷

Algebra

Examples:

#1 $2 + x = 5$

$2 + x = 5$

$x = 5 - 2$

sign change

#2 $(2x+3)\times 5 = 20$

$(2x+3)\times 5 = 20$

$2x + 3 = \frac{20}{5}$

$2x = 4 - 3$

$2 \times x = 1$

$x = \frac{1}{2}$

Examples:

#3 $$\frac{2x-5}{x+3} = \frac{5}{2}$$

$$\frac{2x-5}{x+3} = \frac{5}{2}$$

$$2x-5 = \frac{5}{2} \times \frac{(x+3)}{1}$$

$$2x-5 = \frac{5\times(x+3)}{2}$$

$$2\times(2x-5) = 5\times(x+3)$$

$$4x-10 = 5x+15$$

$$-5x+(4x-10) = 15$$

$$-x-10 = 15$$

$$-x = 15+10$$

$$-x = 25$$

$$x = -25$$

Another way to look at moving numbers in an equation:

We can do same arithmetic operation on both sides of an equation.

Example : 1

$$2x+3=5$$

Subtract 3 from both sides

$$2x=2$$

Divide both sides by 2

$$x=1$$

Example : 2

$$\frac{x}{2}+3=5x$$

Multiply both sides by 2

$$x+6=10x$$

Subtract x from both sides

$$6=9x$$

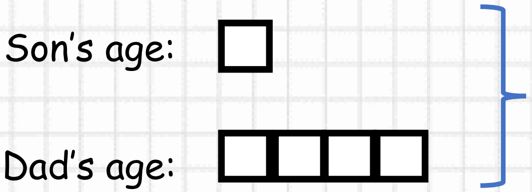
Divide both sides by 9

$$\frac{6}{9}=x$$

$$x=\frac{2}{3}$$

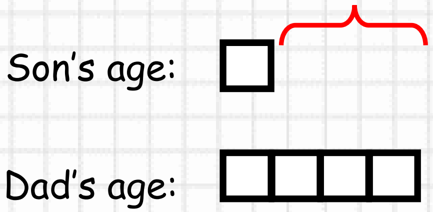
Solving Algebra problem using Bar Model method:

The dad’s age is 4 times the son’s age. If we add 24 to son’s age we get Dad’s age. How old is the dad ?.



Dad’s age is 4 times the son’s age.

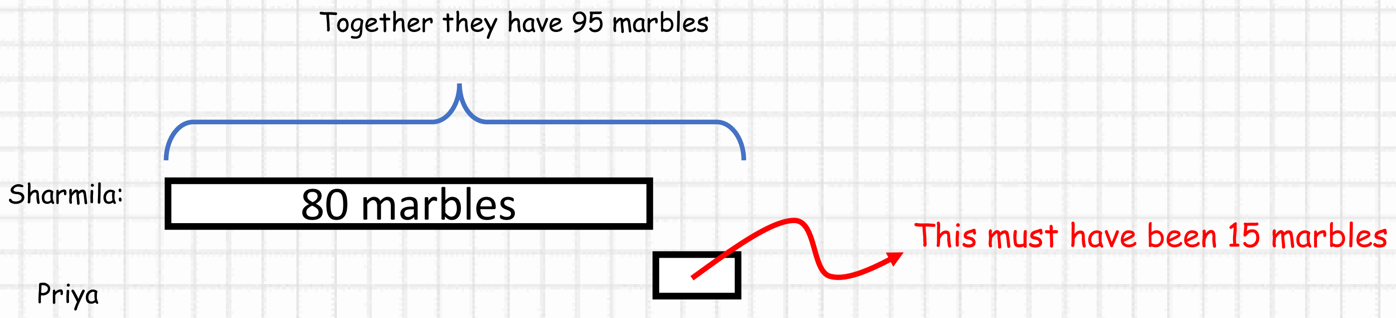
This must be 24 because it is given that the difference between dad’s age and son’s age is 16.

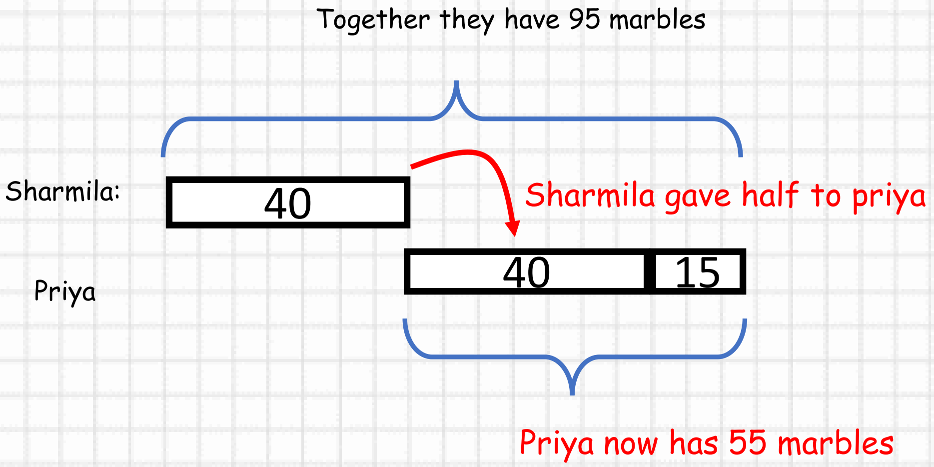


Since 3 blocks = 24, one block =8.
So Dad’s age = 4 blocks = 4*8 = 32

Solving Algebra problem using Bar Model method:

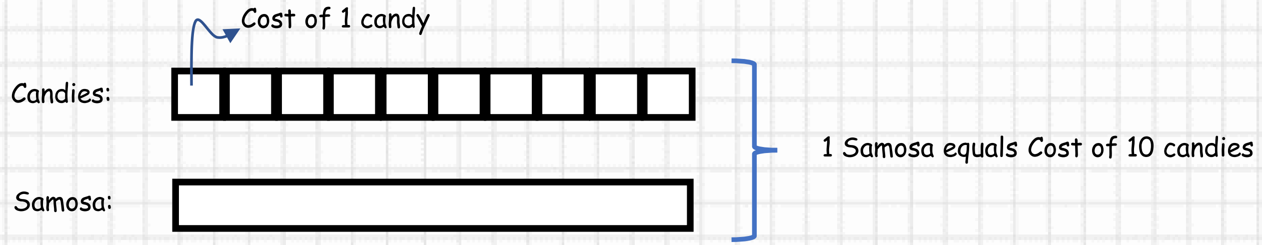
Sharmila has 80 marbles. She gives half of them to Priya. Together they have 95 marbles. How many does Priya have now ?

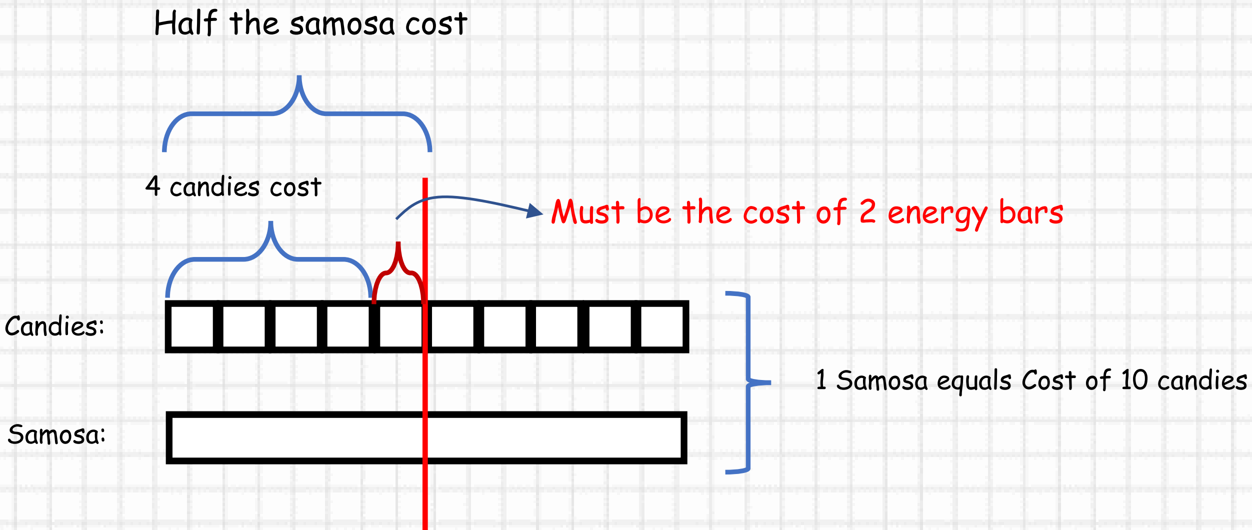




Solving Algebra problem using Bar Model method:

A samosa is 10 times the cost of a candy. If we buy 4 candies and 2 energy bars it is equal to half the cost of a samosa.
If a candy costs Rs 2, how much does one energy bar cost ?.





Cost of 2 energy bars = cost of one candy.
So cost of 1 energy bar = half the candy cost = (1/2)*2 = Rs 1.

The dad's age is 4 times the son's age. If we add 24 to son's age we get Dad's age. How old is the dad ?.

Son's age : s

Dad's age : $d = 4 \times s$ — 1

$s + 24 = d$ — 2

Put 1 in 2

$s + 24 = 4s$

$3s = 24$

$s = 8.$

Dad's age $d = 4s = 4 \times 8 = 32$

Solving Algebra problem using regular method:

Sharmila has 80 marbles. She gives half of them to Priya. Together they have 95 marbles. How many does Priya have now ?

Sharmila has 80 marbles

Together they have 95 marbles

So priya's marbles $p = 95 - 80 = 15$

Sharmila gives half of her marbles to priya

$\frac{1}{2} \times 80 = 40$ marbles to priya

Priya's marbles $= 15 + 40 = 55$ marbles

Solving Algebra problem using regular method:

A samosa is 10 times the cost of a candy. If we buy 4 candies and 2 energy bars it is equal to half the cost of a samosa. If a candy costs Rs 2, how much does one energy bar cost ?.

candy cost : c

samosa cost : $s = 10c$ — ①

$4c + 2e = \frac{s}{2}$ — ②

cost of energy bar

Use ① in ②

$4c + 2e = \frac{10c}{2}$

$4c + 2e = 5c$

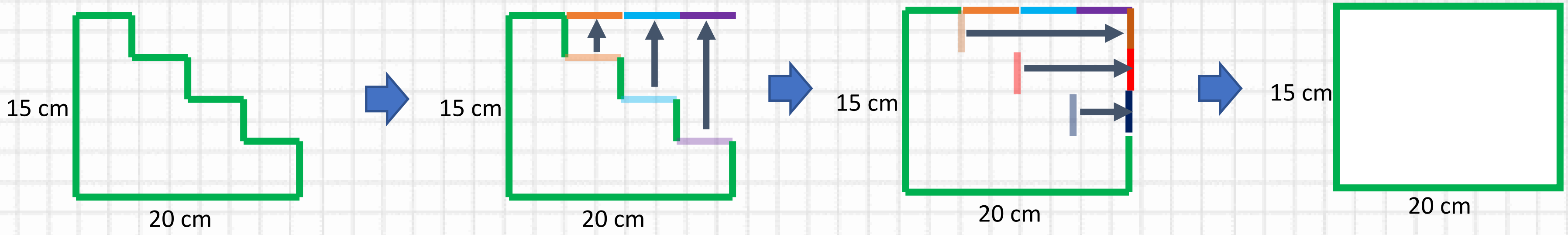
$2e = c$

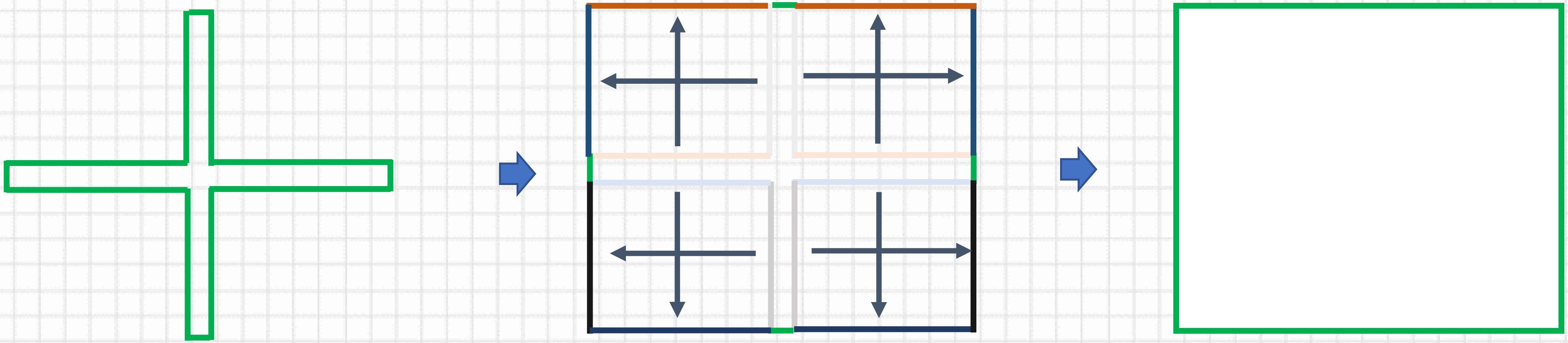
$e = \frac{c}{2}$

Given $c =$ Rs 2. So $e = \frac{\text{Rs } 2}{2} =$ Rs 1

Perimeter and Area

Perimeter:

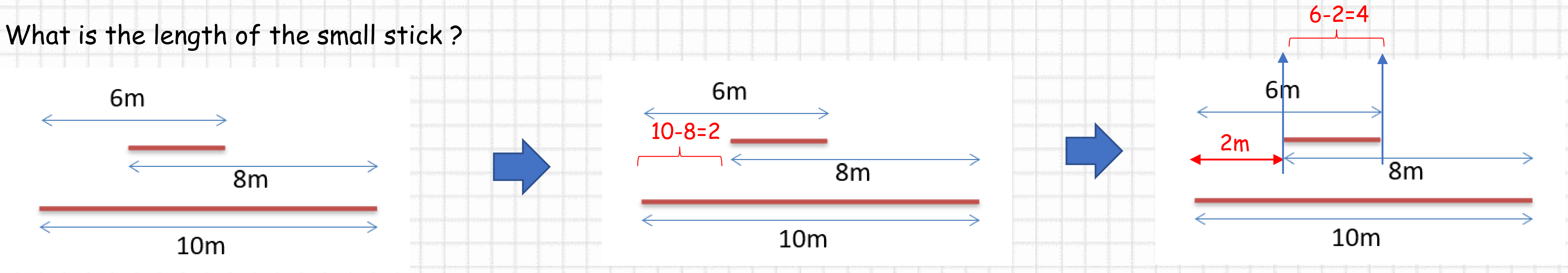




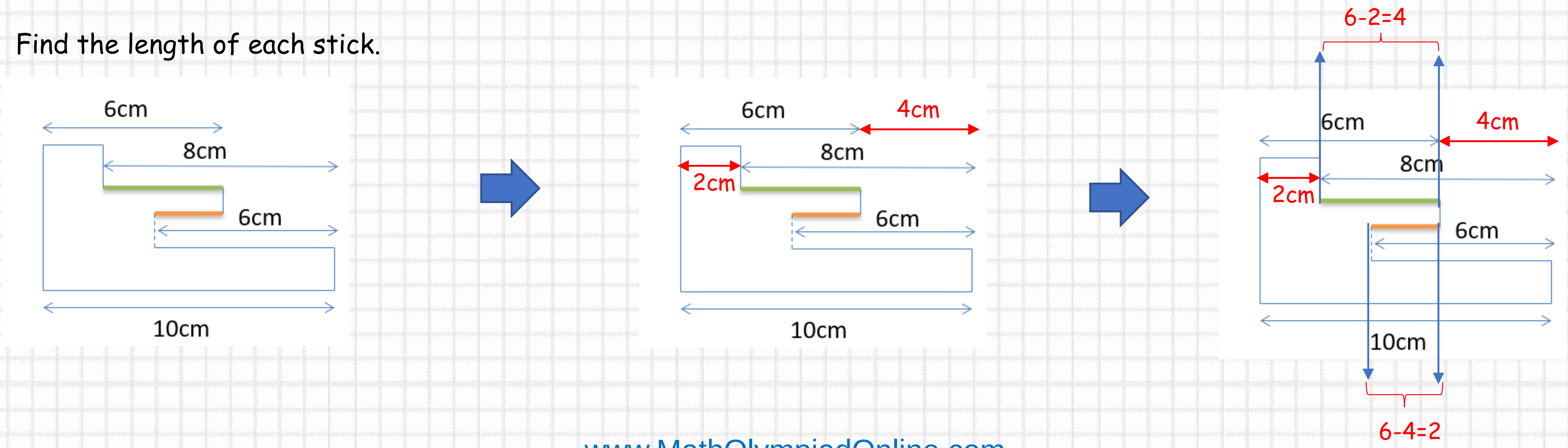
We can transform a given shape to a more regular shape by moving some lines and then find the perimeter for the new shape.

Finding a length based on other measurements.

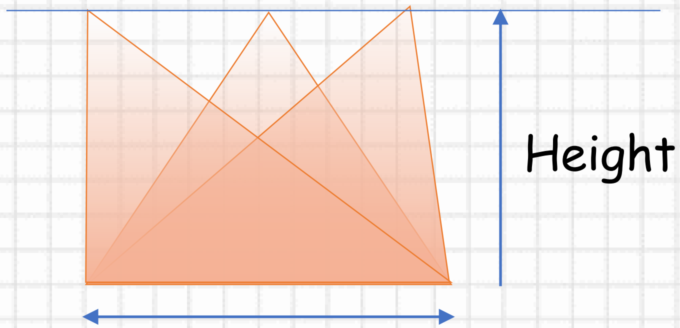
What is the length of the small stick ?



Find the length of each stick.



Area:



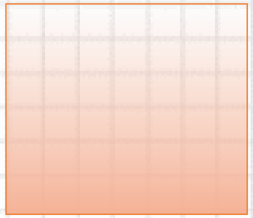
Height = "h"

Base="b"

Area = $\frac{1}{2} * b * h$

All these triangles have the same area

a



a

Area = a*a

If the sides of a square are doubled, the area goes up 4 times.
a→ 2a .. Then area = (2a)*(2a) = 4*a*a which is 4 times the previous area.

Width="w"

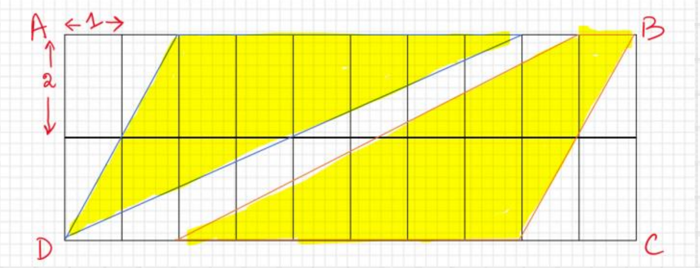
Length = "l"

Area = w*l

Area of a rectangle is twice the area of a triangle with base and height equal to length and width of the rectangle.

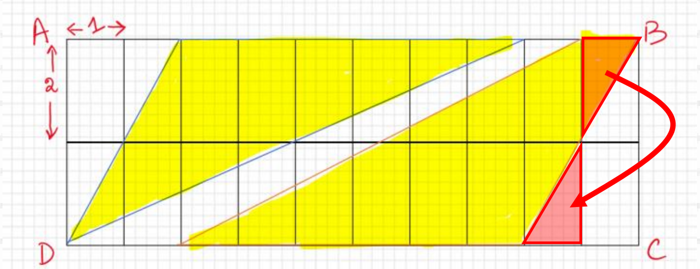
Area:

One interesting type of problem where area can be used to solve easily:



What fraction of rectangle ABCD made of smaller rectangles is not shaded ?

We can try moving some shaded areas to fit elsewhere to make the shaded area more rectangular and count the number of rectangles. But that is hard to do here. Instead we use the idea of area of triangle



Move this here to make the shaded area triangular.

Now we can use the area formula to calculate area of triangles.
The left side shaded triangle has base length of 6 units. Height is 4 units. Area is (1/2)*6*4= 12 square units.
The right side shaded triangle (after moving a portion as shown above) area = (1/2)*7*4 = 14 square units.
Total shaded area = 12+14=26 square units.
Total area of large rectangle = 10*4 = 40 square units.
Therefore fraction unshaded = (40-26)/40 = 14/40 = 7/20.

Ratio and Proportion

Math Olympiad Online
Concept Sheet

$a:b$

antecedant consequent

Meaning of ratio: Comparing two things using division.

Two things X and Y are in ratio a:b means that X is always (a/b) times more than Y. For example, if X is of quantity a, then Y is of quantity b.

Just like fractions, ratio can be scaled up or down by multiplying both terms (antecedent and consequent) or dividing both terms by any value. It doesn't change the comparison between the two quantities.

Example: In a milk factory, Milk and water are mixed in the ratio 5:1.

Milk : Water
5 : 1 ➡ Quantity of milk is always 5 times the quantity of water.

If we take 1litre of water to mix, we should mix it with 5 litres of milk. If we take 10 litres of water to mix, we should mix it with 50 litres of milk.

The ratio 5:1, 50:10 are all equivalent. Such ratios are said to be in "proportion".

Two ratios that are in proportion are expressed as shown below:

extremes

a:b :: c:d

means

$$\frac{a}{b} = \frac{c}{d} \qquad ad = bc$$

Product of Means = Product of Extremes

Example: In a class, all students are given candies and biscuits in the ratio 3:5. A student, Ramu got 25 biscuits. Another student Kumar got 30 biscuits. How many candies did Ramu and Kumar get ?.

Since all students are given candies and biscuits in the same ratio 3:5, the quantities they got must all be in proportion.

For Ramu:
3:5 :: x:25 => x*5 = 3*25 => x = 15 … Ramu must have gotten 15 candies.

For kumar:
3:5 :: y:30 => y*5 = 3*30 => y = 18 … Kumar must have gotten 18 candies.

Solving Problems using Ratio and Proportion:

Cost of Table and Chair are in the ratio 7:3

Table : Chair
7 : 3

Think of the above ratio as :
IF Table costs Rs 7, the chair will cost Rs 3.
This does not mean the table actually costs Rs7. But simply gives a comparative cost of the two items.
For example, IF Tables costs Rs 7*30 = Rs 210, the chair will then cost Rs 3*30 = Rs 90.
IF Table costs Rs 7*50.5 = Rs 353.5, the chair will then cost Rs 3*50.5 = Rs 151.5
and so on.
Now, we can also consider the total cost, the difference cost etc.

Table : Chair	Total Cost	Table cost – Chair cost
7 : 3	10	4

Let's say someone buys equal numbers of tables and chairs and the total cost comes to Rs 5000.

Table : Chair	Total Cost	Table cost – Chair cost
7 : 3	10	4
	$\downarrow$ ×500	
	5000	

Solving Problems using Ratio and Proportion:

Table : Chair	Total Cost	Table cost – Chair cost
7 : 3	10	4
×500 ×500	×500	
3500 1500	5000	

Since the ratio can be scaled up or down and since the total is scaled up by 500 times, the other quantities will also have to be scaled up (multiplied) by 500 to get the actual cost of tables and chairs.

So we can now say that the person must have spent Rs 3500 on tables and Rs 1500 on chairs.

Now, in the same problem, instead of total cost, if the difference in the cost of table and chair in the final purchase is given as Rs 1000, then we work it out as follows.

Table : Chair	Total Cost	Table cost – Chair cost
7 : 3	10	4
×250 ×250		×250
1750 750		1000

Solving Problems using Ratio and Proportion:

Table : Chair	Total Cost	Table cost – Chair cost
7 : 3	10	4
×250 ×250		×250
1750 750		1000

We can conclude that the total purchase was for Rs 1750 + Rs 750 = Rs 2500. And the total cost of tables and chairs are Rs 1750 and Rs 750 respectively.

Geometry

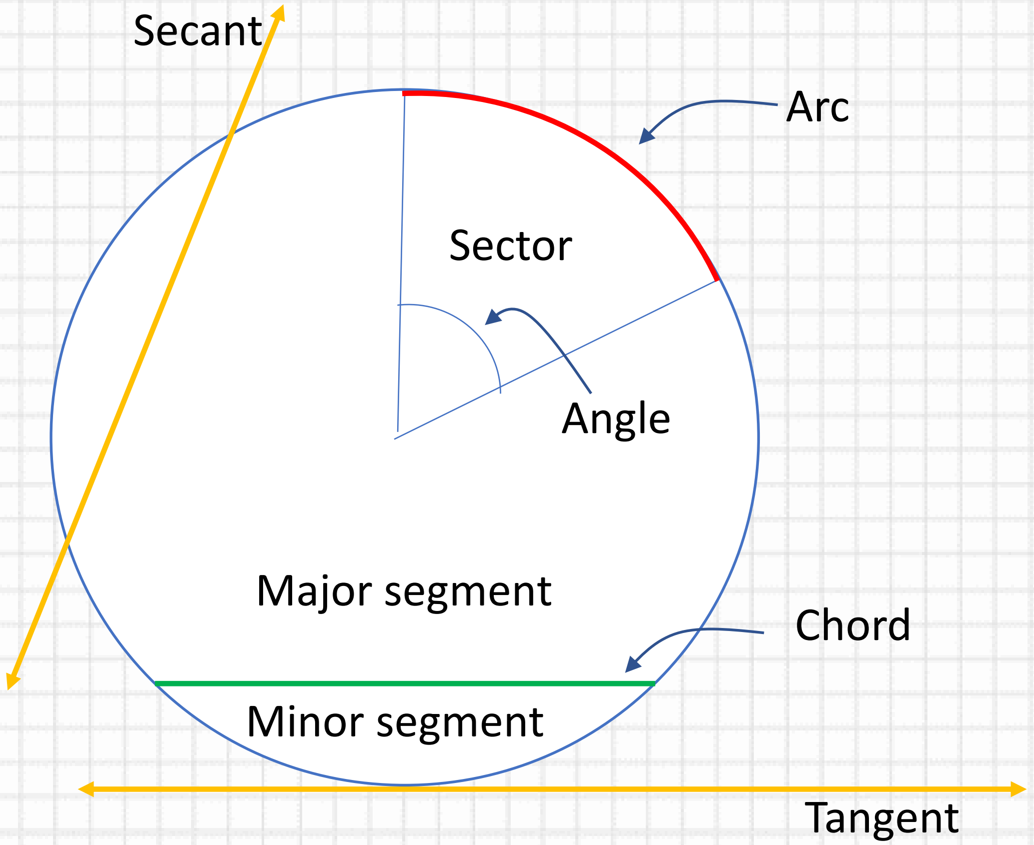
Math Olympiad Online
Concept Sheet

Line — A B — Extends to infinity both sides. Line AB = Line BA

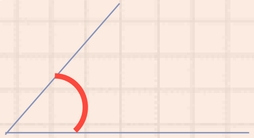
Line Segment — A B — Has finite length. Line AB = Line BA

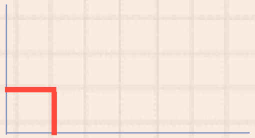
Ray — A B — Starts at A, goes from A to B. extends to infinity on one side. Ray has a direction. Ray AB is not same as Ray BA.

Circle



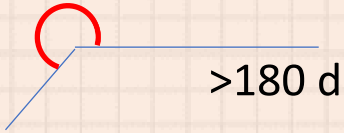
Angle:


<90 degrees → Acute Angle

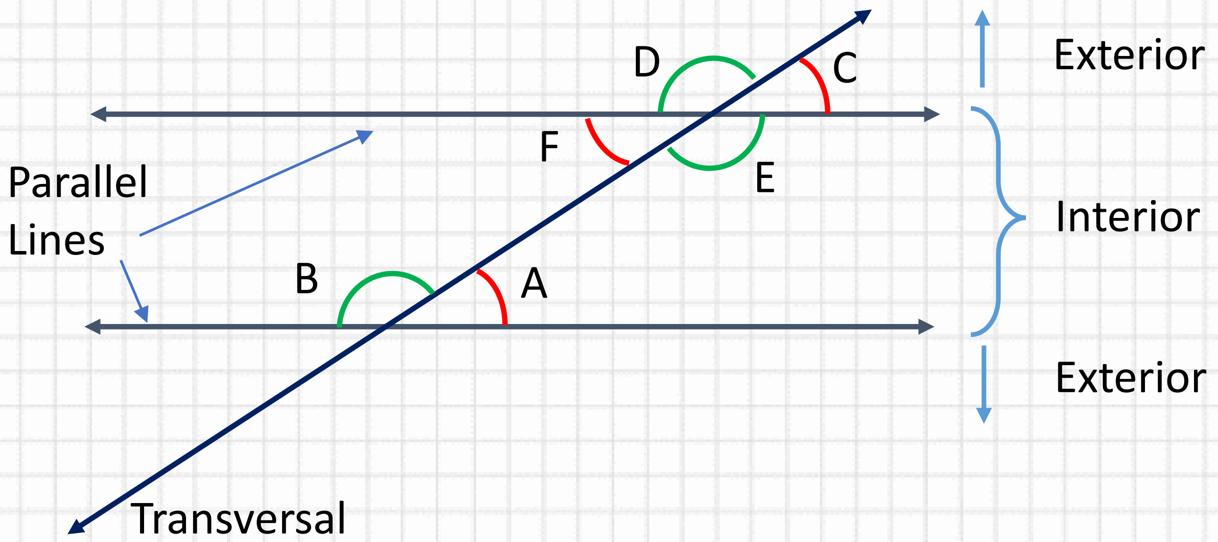

=90 degrees → Right Angle


>90 degrees → Obtuse Angle


=180 degrees → Straight Angle


>180 degrees → Reflex Angle

Parallel Lines: Lines that never meet are called parallel lines.



1. Straight angle : A + B =180 degrees

   A and B are called supplementary angles.

2. Angle A and C made by transversal with the two parallel lines are equal

   A and C are called corresponding angles.

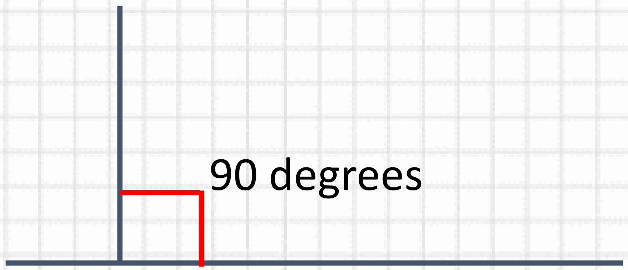
3. Angles C and F are equal.

   C and F are called vertical angles.

From 2 and 3, we can see that Angle A=Angle F

A and F are called interior opposite angles.

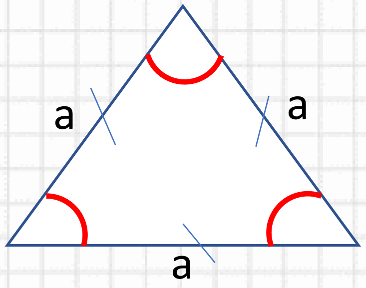
Perpendicular Lines:



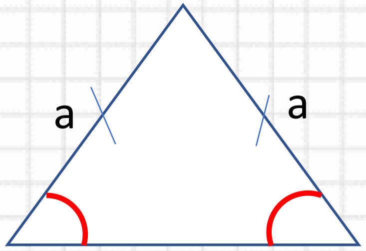
Triangle: 3 sides

Sum of all interior angles = 180 degrees

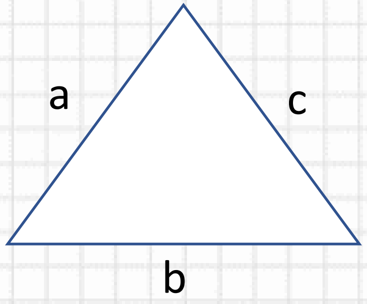
All angles equal <==> All sides equal : **Equilateral** Triangle



Two angles equal <==> Two corresponding sides equal : **Isosceles** Triangle



All angles are different <==> All sides are different : **Scalene** Triangle

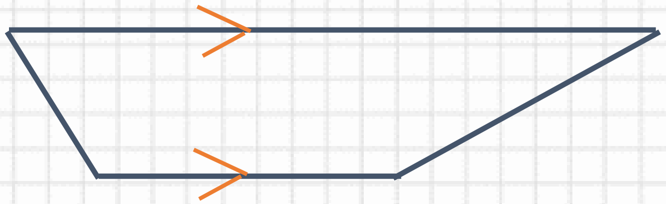


All angles < 90 degrees → Acute angled triangle
One angle = 90 degrees → Right angled triangle
One angle > 90 degrees → Obtuse angled triangle

Quadrilateral: 4 sides

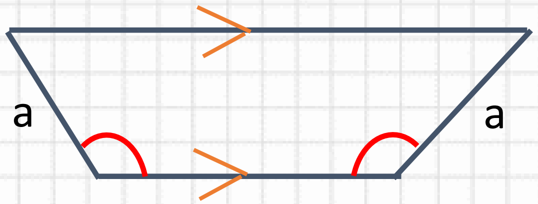
Sum of all interior angles = 360 degrees

1. Two sides are parallel:



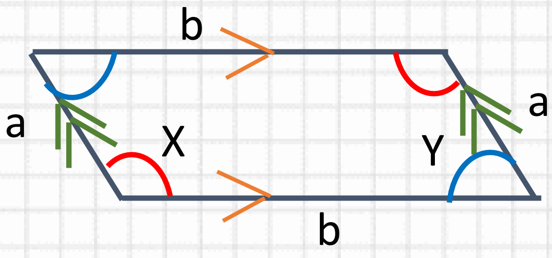
Trapezium

2. Two sides are parallel. Adjacent angles are equal.



Isosceles Trapezium

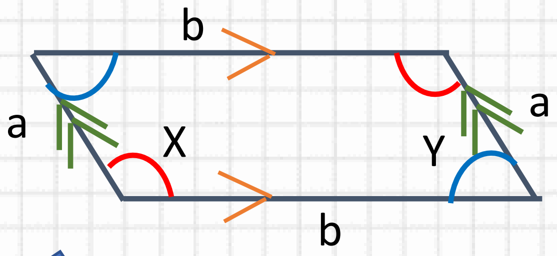
3. All opposite sides are parallel



Parallelogram

X+Y=180 degrees
Interior opposite angles are equal

Quadrilateral: 4 sides

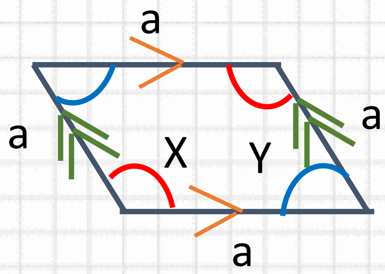


Parallelogram

X+Y=180 degrees

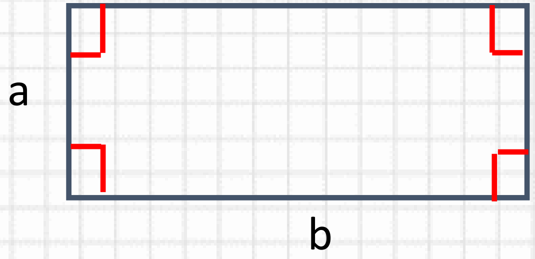
Interior opposite angles are equal

4. In a parallelogram, if all sides are made equal



Rhombus

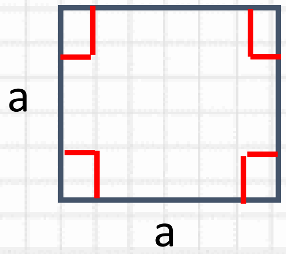
5. In a parallelogram, if adjacent angles are made equal



Rectangle

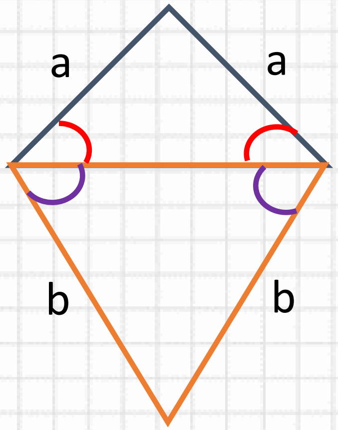
X=Y only if X=Y=90 degrees

6. A rectangle with all sides equal



Square

7. Two pairs of adjacent sides are equal.



Kite

Two isosceles triangles put together with common base

More Polygons: There are more shapes like pentagon, hexagon etc as we increase the number of sides

The General Rule

Each time we add a side (triangle to quadrilateral, quadrilateral to pentagon, etc), we **add another 180°** to the total:

			If it is a **[Regular Polygon](#)** (all sides are equal, all angles are equal)	
Shape	**Sides**	**Sum of Interior Angles**	**Shape**	**Each Angle**
[Triangle](#)	3	180°		60°
[Quadrilateral](#)	4	360°		90°
[Pentagon](#)	5	540°		108°
[Hexagon](#)	6	720°		120°
Heptagon *(or Septagon)*	7	900°		128.57...°
Octagon	8	1080°		135°
Nonagon	9	1260°		140°
...	...	..	...	...
Any Polygon	**n**	(**n**−2) × 180°	n	(**n**−2) × 180° / **n**

Polygon

Regular Polygon

All sides are equal and
All angles are equal

Examples:
Equilateral triangle
Square
Equilateral Pentagon
Equilateral Hexagon

Ir-regular Polygon

Otherwise

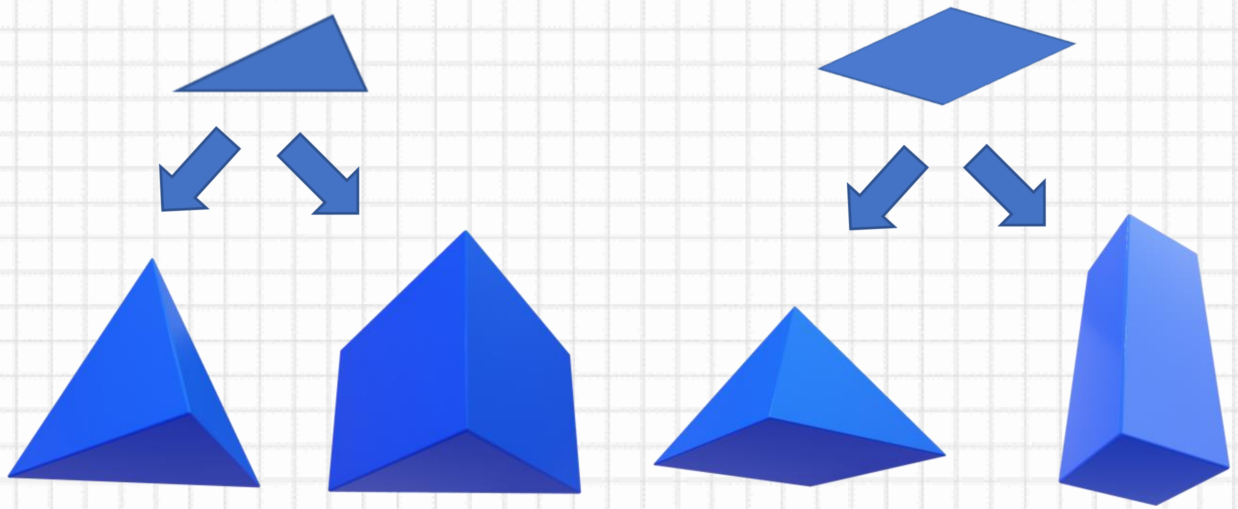
Polyhedron: 3-dimensional polygon is called a polyhedron.

Start from a base polygon

Lift up the base polygon : There are two ways to lift it up…

1) Lift it up so that it gets to a point on top → Pyramid

2) Lift it up so the base polygon shape is maintained all the way as we lift it → Prism



(Tetrahedron)

Vertex: Corner points

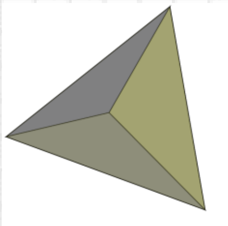
Edge: The line segments connecting certain pairs of vertices

Face: The surface formed by two dimensional polygons

Other Polyhedra:

n	polyhedron
4	tetrahedron
5	pentahedron
6	hexahedron
7	heptahedron
8	octahedron
9	nonahedron
10	decahedron
11	undecahedron
12	dodecahedron
14	tetradecahedron
20	icosahedron
24	icositetrahedron
30	triacontahedron
32	icosidodecahedron
60	hexecontahedron
90	enneacontahedron

tetrahedron

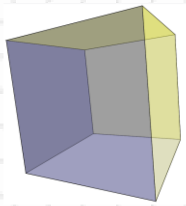


4 polygon faces

pentahedron

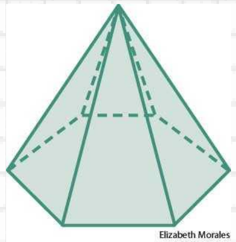
5 polygon faces

hexahedron



6 polygon faces

heptahedron



7 polygon faces

Polyhedra: Relation between Faces, Vertices and Edges

Counting Faces, Vertices and Edges

When we count the number of faces (the flat surfaces), vertices (corner points), and edges of a polyhedron we discover an interesting thing:

The number of **faces**
plus the number of **vertices**
minus the number of **edges** equals **2**

This can be written neatly as a little equation:

$$F + V - E = 2$$

It is known as Euler's Formula (or the "Polyhedral Formula") and is very useful to make sure we have counted correctly!

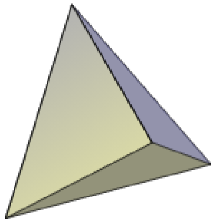
Regular Polyhedra: A polyhedra where each face is the same regular polygon and at each vertex the same number of polygons meet.
Regular polyhedral are also called as Platonic solids. There are only 5 such shapes

The Platonic Solids

For each solid we have two printable nets (with and without tabs). You can make models with them!

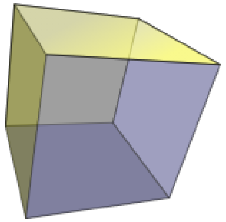
Print them on a piece of card, cut them out, tape the edges, and you will have your own platonic solids.

Tetrahedron



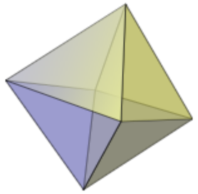
- 3 triangles meet at each vertex
- 4 Faces
- 4 Vertices
- 6 Edges
- Tetrahedron Net
- Tetrahedron Net (with tabs)
- Spin a Tetrahedron

Cube



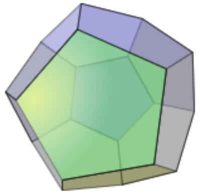
- 3 squares meet at each vertex
- 6 Faces
- 8 Vertices
- 12 Edges
- Cube Net
- Cube Net (with tabs)
- Spin a Cube

Octahedron



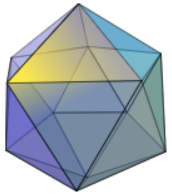
- 4 triangles meet at each vertex
- 8 Faces
- 6 Vertices
- 12 Edges
- Octahedron Net
- Octahedron Net (with tabs)
- Spin an Octahedron

Dodecahedron



- 3 pentagons meet at each vertex
- 12 Faces
- 20 Vertices
- 30 Edges
- Dodecahedron Net
- Dodecahedron Net (with tabs)
- Spin a Dodecahedron

Icosahedron



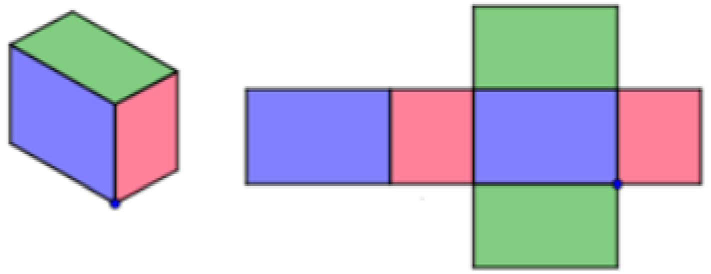
- 5 triangles meet at each vertex
- 20 Faces
- 12 Vertices
- 30 Edges
- Icosahedron Net
- Icosahedron Net (with tabs)
- Spin an Icosahedron

Non-Polyhedron: 3-dimensional shapes where some faces are not made of polygons

Example: Cone, Cylinder

Net of a Solid:

Net of Rectangular Prism



Net of Cylinder